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Analysis of a three-component impedance using two sine waves

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Abstract* – A common method to identify several components modeling a complex impedance is by impedance spectroscopy. The unknown impedance is measured by applying several sine waves and the impedance components are determined by curve-fitting to the results obtained at each measurement frequency. This paper presents a novel method able to determine three independent impedance components by using only two sine waves instead of the customary three sine waves that would be required in impedance spectroscopy. Each sine wave yields two measurement results: one in phase with the injected signal and another one in quadrature (90° phase shift) with that signal. Using two sine waves yields four measurement results and solving the corresponding equation system would permit us to calculate up two four independent impedance components. We have derived the equations needed to calculate three independent components and tested the method by measuring an impedance network consisting of three components using a commercial impedance analyzer. The resulting error is less than 1.3 % for each component. The proposed method outperforms common impedance spectroscopy in measurement time, calculation power required, and instrumentation simplicity.

I. INTRODUCTION

Impedance-based sensors are often modeled by several lumped circuits components whose impedance depends on the measurand [1]. Simple impedance models consist of a single circuit component such as a resistance or a capacitance. More advanced models include two components connected either in parallel (Fig. 1*a*) or series (Fig. 1*b*). However, measuring that impedance at several frequencies often yields component values that do not remain constant with frequency [2]. Models including three constant-value components may then be able to describe the sensor impedance in a broader frequency range than two-component models. Cell-electrolyte impedance based on the simplified Randles cell [3] model, for example, can be described by the series combination of a resistance and a capacitance shunted by another resistance (Fig. 1*c*). In spite of its simplicity, or perhaps because of it, this three-component model is quite common in electrochemical impedance spectroscopy and it is also used in bioimpedance measurements based on either sine-wave [4] or step-voltage [5] excitation.

This paper describes a novel method to determine the three impedance components in Fig. 1*c* by using only two sine waves. Three independent components can be determined from at least three independent measurements. Normally, each single measurement is obtained by injecting a sine wave of a given frequency. Here we propose to calculate two components at each measurement frequency by considering the model in Fig. 1*b* and detecting the in-phase, and quadrature components with respect to the injected sine wave. Therefore, two frequencies yield two sets of two impedance components, and from them it is possible to calculate the component values in Fig. 1*c* that model the actual impedance. The method has been tested by a commercial impedance analyzer able to measure impedances modeled as in Fig. 1*b* but unable to directly determine three impedance components as in Fig. 1*c*.

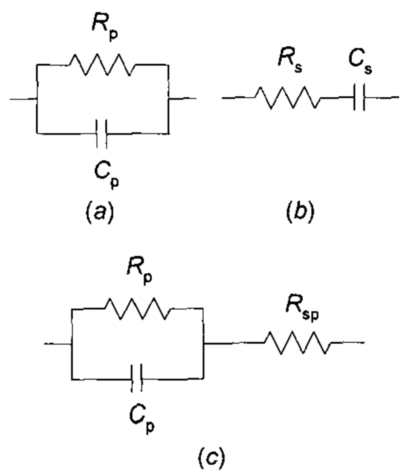

Fig. 1. Impedance sensors are better modeled by two or three independent components rather than a single component.

II. EQUIVALENT IMPEDANCE CIRCUITS

The equivalent impedance of the network in Fig. 1*c* is

$$Z(j\omega)=\left(R_s+\frac{R_p}{1+\omega^2C_p^2R_p^2}\right)-j\frac{\omega C_pR_p^2}{1+\omega^2C_p^2R_p^2} \qquad (1)$$

that can be described by the series connection of a resistance and a capacitance according to

$$Z(j\omega)=R_{EQ}(\omega)+\frac{1}{j\omega C_{EQ}(\omega)} \qquad (2)$$

where

$$R_{EQ} = R_s + \frac{R_p}{1+\omega^2\tau^2} \tag{3}$$

$$C_{EQ} = \frac{1+\omega^2\tau^2}{\omega^2\tau R_p} \tag{4}$$

and

$$\tau = C_p R_p \tag{5}$$

Note that both the equivalent resistance and capacitance depend on frequency, as corresponds to a three-constant-component impedance modeled by only two components.

Measuring the impedance network in Fig. 1*c* at two frequencies f_1 and f_2 by an impedance analyzer that models the unknown impedance as in Fig. 1*b*, yields R_1 and C_1 at f_1, and R_2 and C_2 at f_2. From (3), the measured resistances yield two equations ($R_{EQ} = R_1$ at f_1, and $R_{EQ} = R_2$ at f_2) and, from (4), the measured capacitances yields two additional equations ($C_{EQ} = C_1$ at f_1, and $C_{EQ} = C_2$ at f_2). Solving those four equations we obtain

$$\tau = +\sqrt{\frac{\omega_1^2 C_1 - \omega_2^2 C_2}{\omega_1^2\omega_2^2(C_2 - C_1)}} \tag{6}$$

$$R_p = (R_1 - R_2)\frac{1+(\omega_1^2+\omega_2^2)\tau^2 + \omega_1^2\omega_2^2\tau^4}{\tau^2(\omega_2^2-\omega_1^2)} \tag{7}$$

$$C_p = \frac{\tau}{R_p} \tag{8}$$

$$R_{sp} = R_1 - \frac{R_p}{1+\omega_1^2\tau^2} \tag{9}$$

where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, and only the positive solution has been considered for τ because time constants are positive.

III. MATERIALS AND METHODS

The proposed method has been tested by building a test impedance like that in Fig. 1*c* by connecting a carbon-film resistor R_{sp} = 330 Ω (5 % tolerance, 0.25 W) in series with a polyester capacitor C_p = 220 nF (CPM-N BH014D0224k AVX) shunted by another carbon-film resistor R_p = 1.2 kΩ (5 % tolerance, 0.25 W). The impedance analyzer was Agilent model 4294A, whose typical uncertainty in the measurement ranges and frequencies used is about 0.3 %. The measurement frequencies were 900 Hz and 1 kHz, and the applied voltage was 100 mV. Applying 1 V (peak-to-peak) to each separate component did not show any nonlinearity.

In order to later validate the method, we have first measured each individual component at the selected frequencies, aiming to assess their frequency dispersion. Next we have measured the test impedance network at those two frequencies, and the respective resistance and capacitance have been substituted in equations (6) to (9) in order to calculate R_{sp}, R_p, and C_p.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Table 1 shows the measured values for the separate components. Tolerance and measurement uncertainty explain the difference between measurement results and nominal values. Both resistors display a constant value in the frequency band tested, but the capacitance and shunting (leakage) resistance of the capacitor depends on the frequency. This dependence is common for commercial capacitors [6]. When building the impedance in Fig. 1*c*, that leakage resistance will affect the apparent value of R_p.

Table I. Measured values for the individual components connected as shown in Fig. 1c using a commercial impedance analyzer (Agilent 4294A).

f	R_{sp}= 330 Ω	R_p= 1.2 kΩ	C_p= 220 nF, AVX CPM-N BH014D0224k
900 Hz	327 Ω ± 1 Ω	1179Ω ± 3.5 Ω	(222.7 nF ± 0.7 nF) || 208 kΩ
1 kHz	327 Ω ± 1 Ω	1179Ω ± 3.5 Ω	(222.5 nF ± 0.7 nF) || 200 kΩ

Table 2 shows the result when the three components are connected as shown in Fig. 1*c* and the impedance analyzer calculates the real and imaginary parts of the impedance according to the model in Fig. 1*b*. Both the serial resistance and the serial capacitance change with frequency, in spite of the closeness of the two measurement frequencies. This reveals that a two component model is not enough to describe the three-component network.

Table II. Results obtained when measuring the network in Fig. 1*c* with an impedance analyzer (Agilent 4294A) that considers the model in Fig. 1*b*.

f	R_s	C_s
900 Hz	696 Ω ± 2 Ω	325 nF ± 1 nF
1 kHz	645 Ω ± 2 Ω	306 nF ± 1 nF

Applying eqns. (6) to (9) to the values in Table 2 yields the component values shown in Table 3. The results are quite close to the actual component values in Table 1. The error with respect to the individual values measured at 1 kHz is less than 1.3 %. The highest error is obtained for R_{sp}. Some possible error sources are: the finite leakage resistance of the capacitor, which according to Table 1 depends on the frequency, rounding in calculations, and the error of the commercial impedance analyzer. Nevertheless, the achieved error is acceptable for most industrial applications.

Table III. Impedance components calculated by the proposed method from the results in Table 2. Errors are relative to the measured values in Table 1, excluding the leakage resistance for the capacitor.

	Calculated value	Measured value	Error
R_{sp}	331 Ω	327 Ω	1.2 %
R_p	1186 Ω	1179 Ω	0.6 %
C_p	223.6 nF	222.5 nF	0.5 %

V. CONCLUSION

We have designed a method to determine three independent components of an impedance network by measuring it with two sine waves of different frequency. For each injected signal we determine the in-phase and quadrature components and then solve the equation system formed by (6) to (9). The method has been tested by measuring an impedance network built from a resistor in series with a capacitor shunted by another resistor. An impedance analyzer determines the real and imaginary parts of the impedance measured at each frequency. Replacing the results and solving the equation yields the three components with less than 1.2 % error. Building an impedance meter relying on the designed method would require only two sine waves and calculating (6) to (9) instead of several sine waves and curve fitting used in conventional impedance spectroscopy.

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